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Normal and Subnormal Series

Def A Subnormal series of a Group G is a chain of Subgroup

$$G = G_0 > G_1 > \text{---} > G_n$$

Such that $G_i > G_{i+1}$ for all i.
If in addition $G \triangleright G_i$ $\forall$ all i then it is called a normal series. The factors of the series are the quotient groups G_i / G_{i+1}. The length of the series is the number of strict inclusions.

Def: A Subnormal Series $G = G_0 > G_1 > \text{--} > G_n = \{e\}$ is a composition series if each factor G_i / G_{i+1} is Simple. It is solvable series if each factor is abelian

Def: A Subnormal Series $G = H_0 > H_1 > \text{---} > H_m$ is a refinement of a Subnormal series $G = G_0 > G_1 > \text{---} > G_n$ if $G_0, G_1, \text{---} G_n$ is a Subsequence of $H_0, H_1, \text{---} H_m$. A refinement is proper if its length is larger than that of the original Series.

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Th 1 Every finite group G has a composition series.

Th2. Every refinement of a Solvable series is a solvable series

Th3 A Subnormal series is a composition series iff it has no proper refinement

Th 4. A Group G is Solvable iff it has a Solvable series.

Th5.) A Group G is Solvable iff it has a composition series whose factors are cyclic of prime order

Def. Two subnormal series S and T of a Group G are equivalent if there is a One-to-One correspondence between the nontrivial factors of S and T Such that corresponding factors are isomorphic.